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学术论文

Knot Optimization for Biharmonic B-splines on Manifold Triangle Meshes
流形三角网格上双调和B样条结点优化

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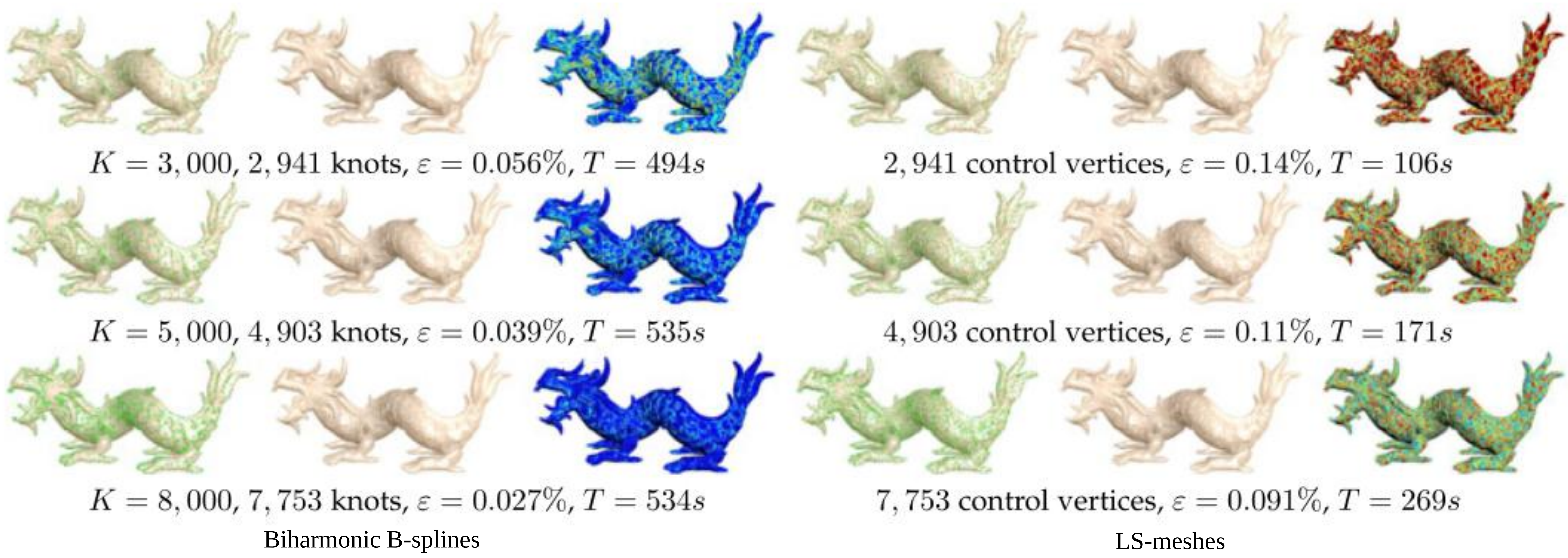

Figure 1: Comparing biharmonic B-splines with LS-meshes in terms of approximation error and runtime performance. Biharmonic B-splines determine the knots by L_1 optimization, whereas LS-meshes choose the control vertices by greedy selection combined with local error maxima. With the same number of knots and control vertices, biharmonic B-splines consistently outperform LS-meshes in terms of approximation error. However, LS-meshes are 2 to 5 times faster to construct than biharmonic B-splines. We visualize the fitting errors using colors, where warm colors indicate large error and cold colors small error.

Biharmonic B-splines, proposed by Feng and Warren [1], are one such novel advancement, that extends univariate B-splines to planar and curved domains. In [2], the key observation is that the discrete bi-Laplacian is a well-behaved analog of divided differences. The flexibility of biharmonic B-splines comes with a high price: (1) The need of Voronoi tessellation is unavoidable; (2) The computation of the distance function is lacking an analytical formulation on general surfaces; (3) The bases must be re-evaluated wherever domain re-configuration is carried out during knot refinement and coarsening; and (4) Bi-Laplacian operators are well defined on simple domains.

To combat the aforementioned difficulties, the overarching goal of this paper is to expand the horizon of biharmonic B-splines at both theoretic and practical fronts. We promote the use of biharmonic B-splines through a novel, yet simpler, and equivalent *implicit* formulation. Rather than the conventional *explicit* formulation. The proposed implicit representation naturally and elegantly brings forth two fundamental properties, which otherwise cannot be easily derived in the explicit representation. We develop a new computational framework for constructing biharmonic B-splines on triangular meshes. Our framework consists of algorithms for spline evaluation, optimal knot selection, data interpolation and hierarchical data decomposition.

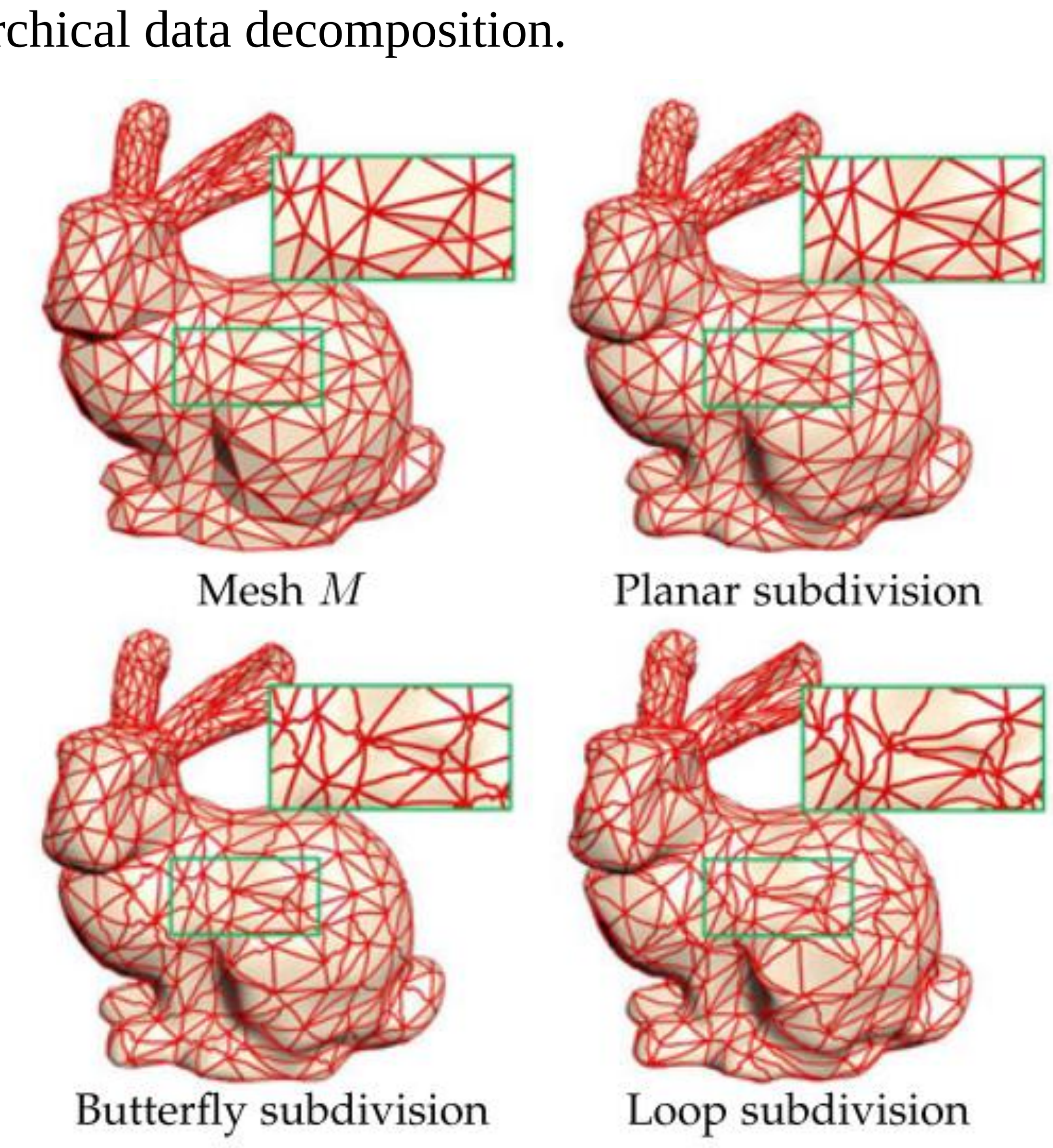

Figure 2. Evaluating the splines using planar subdivision and other subdivision schemes. Only planar subdivision produces correct results. With Loop subdivision and butterfly subdivision, the subdivided mesh M' has different geometry than the domain mesh M. Thus, the value $f(x)$ makes no sense for a point $x \notin M$. The red curves are the images of the edges of M.

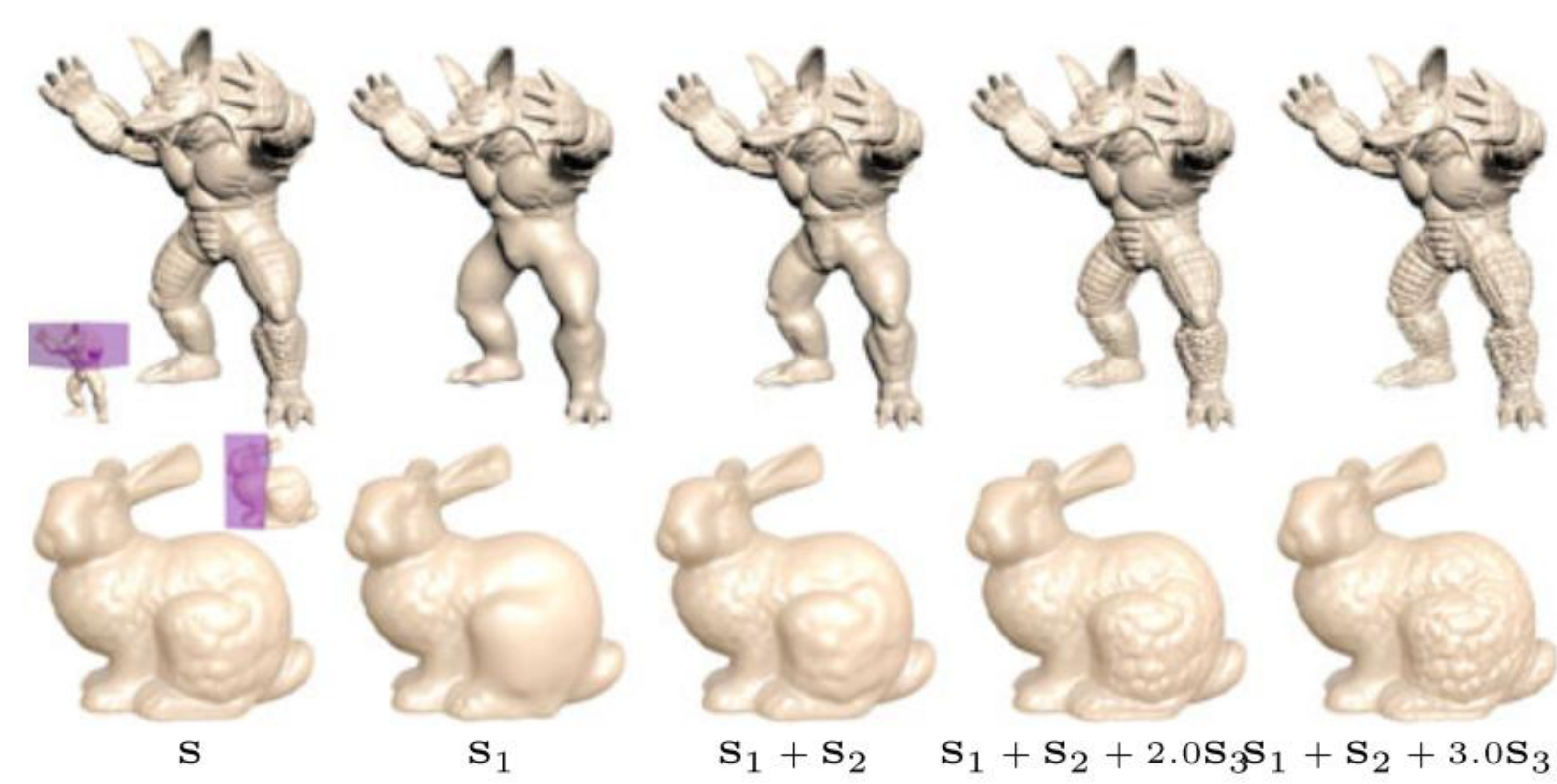

Figure 3. Hierarchical data decomposition.

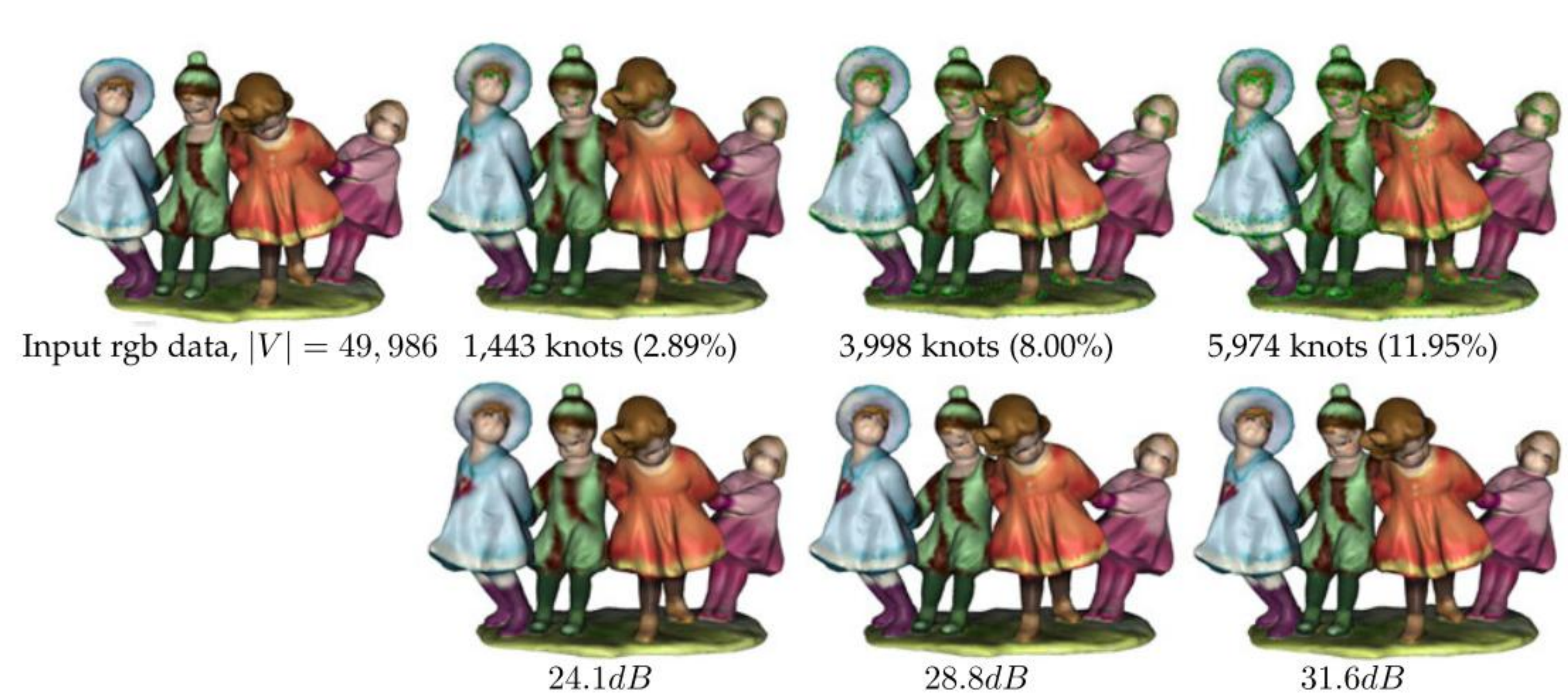

Figure 4. Fitting vector-valued data (i.e., rgb color) defined on a genus-8 model. Top: the input data and the knots. The percentage shows the knot vertex ratio. Bottom: the reconstructed data.

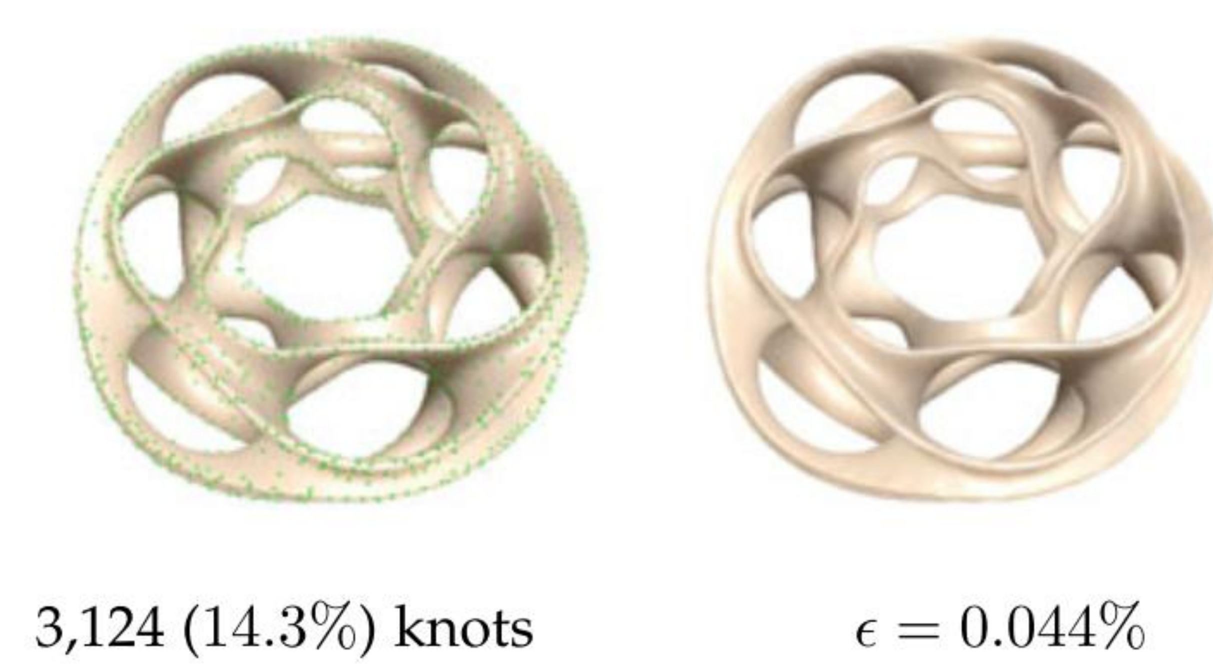

Figure 5. Thanks to its parameterization-and-singularity-free property, biharmonic spline can be easily defined on 3D surfaces of complicated topology, which are quite difficult for conventional splines. The percentage shows the knot-vertex ratio.

Reference:
[1] P. Feng and J. Warren, “Discrete bi-Laplacians and biharmonic B-splines,” ACM Trans. Graph., vol. 31, no. 4, pp. 115:1–115:11, 2012