关键技术

磁流体动力学全隐式模拟

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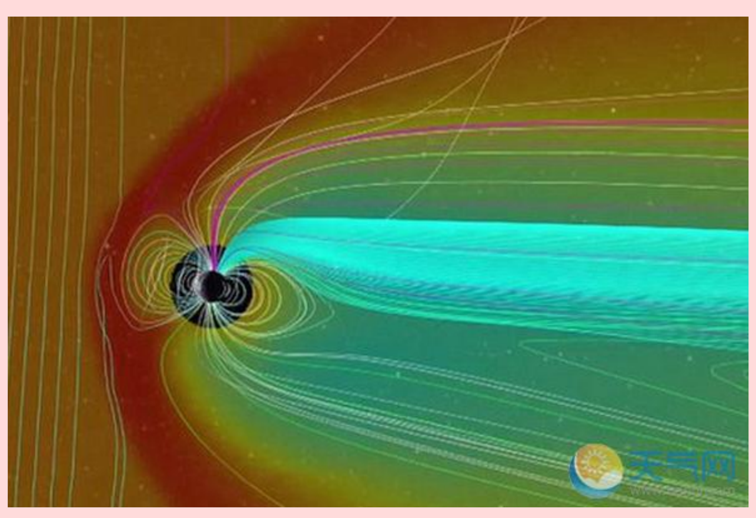
1 背景介绍-磁场重联

- 磁场重联是宇宙空间最有效的能量转换和释放方式，同 时也是造成太阳耀斑，日冕物质抛射，地球磁层亚暴，以及受控热核聚变中磁扰的直接原因。研究磁重联不仅有科学意义，也有应用价值。美国宇航局2015年发射的MMS卫星计划，其主要探测目标就是磁重联。
- 磁场重联可以用电阻型 MHD模型来描述，其中研究比较多的是电阻霍尔磁流体模型。
- 数学上，电阻霍尔磁流体模型（resistive Hall MHD）可以用Navier-Stokes方程耦合Maxwell方程得到。



NASA MMS卫星首次观测到磁场重联

2 Resistive Hall MHD方程

$$nm_e\left(\frac{\partial \mathbf{V_e}}{\partial t}+(\mathbf{V_e}\cdot\nabla)\mathbf{V_e}\right)=-\nabla p_e-ne(\mathbf{E}+\mathbf{V_e}\times\mathbf{B})+\nu_e\nabla^2\mathbf{V_e}+ne\eta\mathbf{j}$$

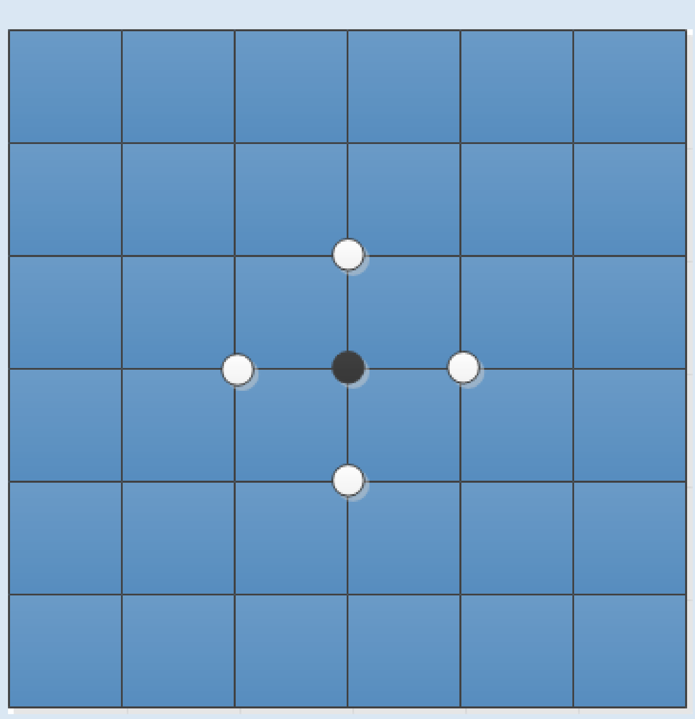
$$nm_i\left(\frac{\partial \mathbf{V_i}}{\partial t}+(\mathbf{V_i}\cdot\nabla)\mathbf{V_i}\right)=-\nabla p_i+ne(\mathbf{E}+\mathbf{V_i}\times\mathbf{B})+\nu_i\nabla^2\mathbf{V_i}-ne\eta\mathbf{j}$$

对于二维问题，其流函数形式入下：

$$\begin{cases} \nabla^2\phi^1 = U^1 \\ \nabla^2\psi^1 = \dfrac{1}{d_e^2}(\psi^1-F^1) \\ \dfrac{\partial U^1}{\partial t}+[\phi^1,U^1] = \dfrac{1}{d_e^2}[F^1,\psi^1]+\nu\nabla^2U^1+\dfrac{1}{d_e^2}\left(\dfrac{\partial\psi^1}{\partial y}F_{eqx}+\dfrac{\partial F^1}{\partial y}B_{eqy}\right) \\ \dfrac{\partial F^1}{\partial t}+[\phi^1,F^1] = \rho_s^2[U^1,\psi^1]+\eta\nabla^2\psi^1+\left(\dfrac{\partial\phi^1}{\partial y}F_{eqx}+\rho_s^2\dfrac{\partial U^1}{\partial y}B_{eqy}\right), \end{cases}$$

3 离散与求解

- 每个网格点4个未知量：φ，ψ，U，F
- 空间：二阶中心差分格式
- 时间：三阶向后差分格式
- 采用Newton-Krylov-Schwarz方法求解



4 Newton-Krylov-Schwarz计算框架

Algorithm 1 Newton-LS算法:$E_{k+1}=E_k-\lambda_k J(E_k)^{-1}G(E_k)$

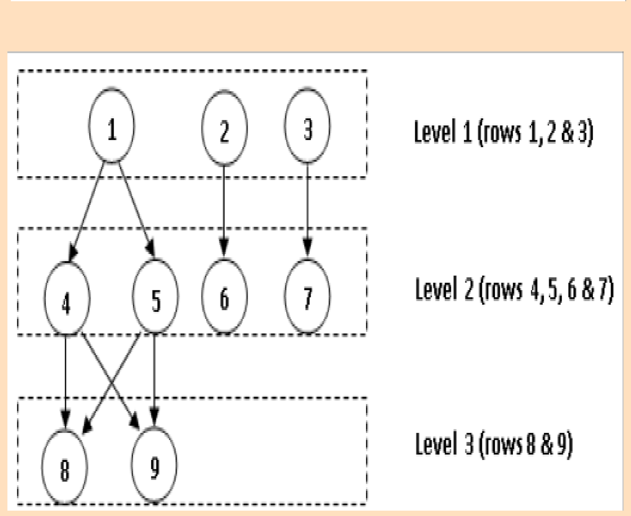
for $i=0,1,...,it_{max}$ **do**
  Compute $G(E_k)$, use Formfunction;
  Compute $J(E_k)$, use FormJacobian;
  Solve the linear system $J(E_kM_k^{-1})s_k=-G(E_k)$,using a preconditioned GMRES;
  Use line search method to get the λ,$E_+=E_k+\lambda s_k$;
  Set $E_{k+1}=E_+$, go to step 1 unless a stopping condition has been met.
end for

Left restricicted additive Schwarz preconditioner

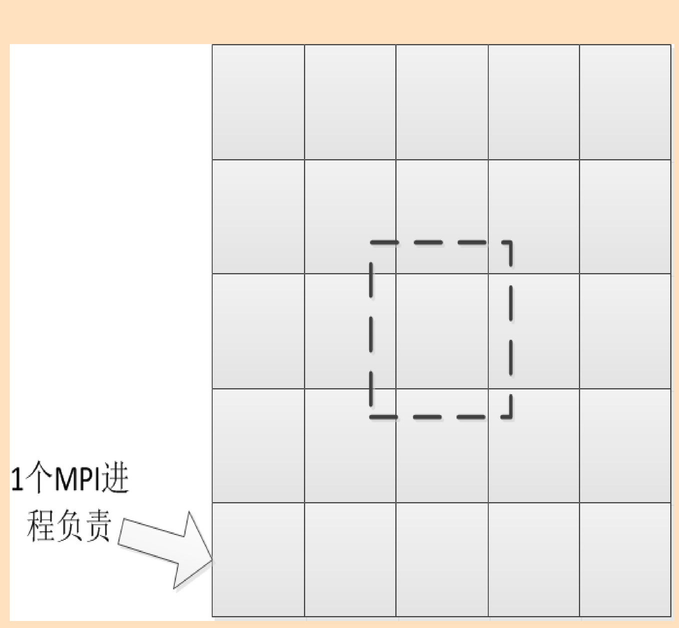
$$M_k^{-1}=\sum_{j=1}^{N}(R_j^0)^TB_j^{-1}R_j^\delta,$$

1: Compute $r_0=M^{-1}(b-Ax_0)$,$\beta=||r_0||_2$,and $v_1=r_0/\beta$
2: For j = 1,2,...,m Do:
3:   Compute $\omega:=M^{-1}Av_j$
4:   For $i=1,...,j$ Do:
5:     $h_{ij}:=(\omega,v_i)$
6:     $\omega:=\omega-h_{ij}v_i$
7:   EndDo
8:   Compute $h_{j+1,j}=||\omega||_2$ and $v_{j+1}=\omega/h_{j+1,j}$
9:   Define $V_m:=[v_1,\cdots,v_m]$,$\bar{H}_m=\{h_{i,j}\}_{1\le i\le j+1;1\le j\le m}$
10: EndDo
11: Compute y_m which minimizes $||\beta e_1-\bar{H}_my||_2$ and $x_m=x_0+M^{-1}V_my_m$
12: If satisfied then Stop,else set $x_0:=x_m$ and GoTo 1

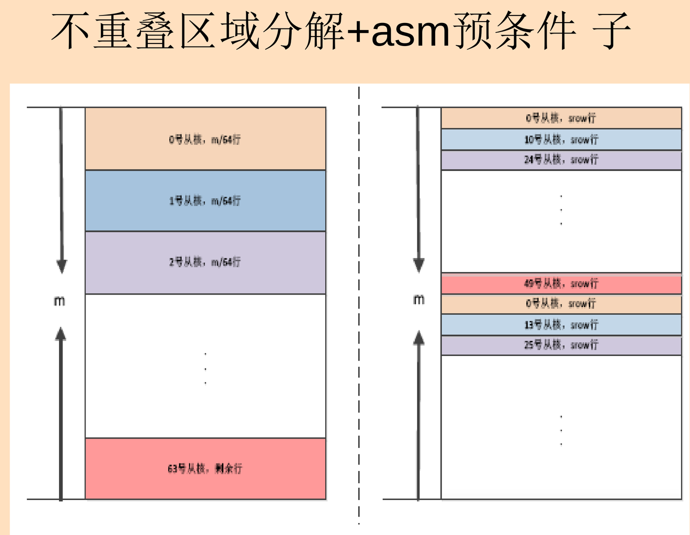
5 实现与优化



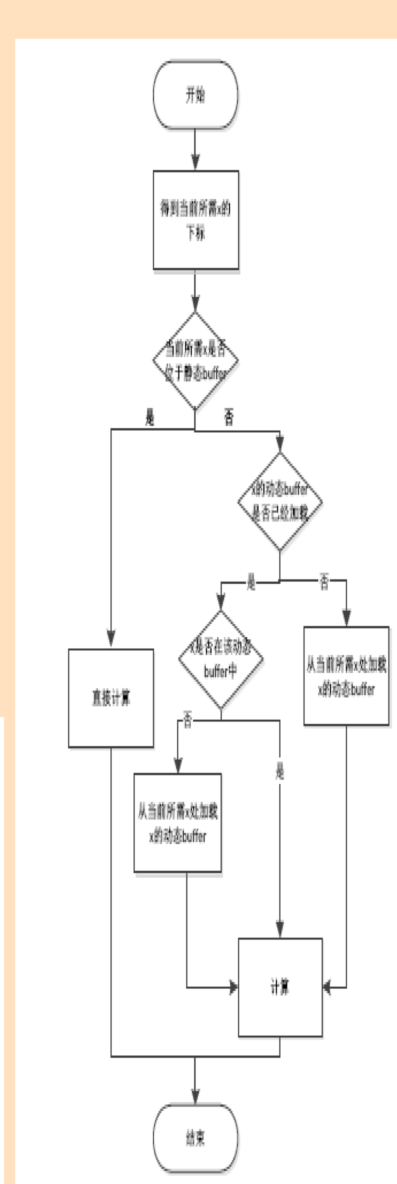
Level Scheduling



不重叠区域分解+asm预条件 子

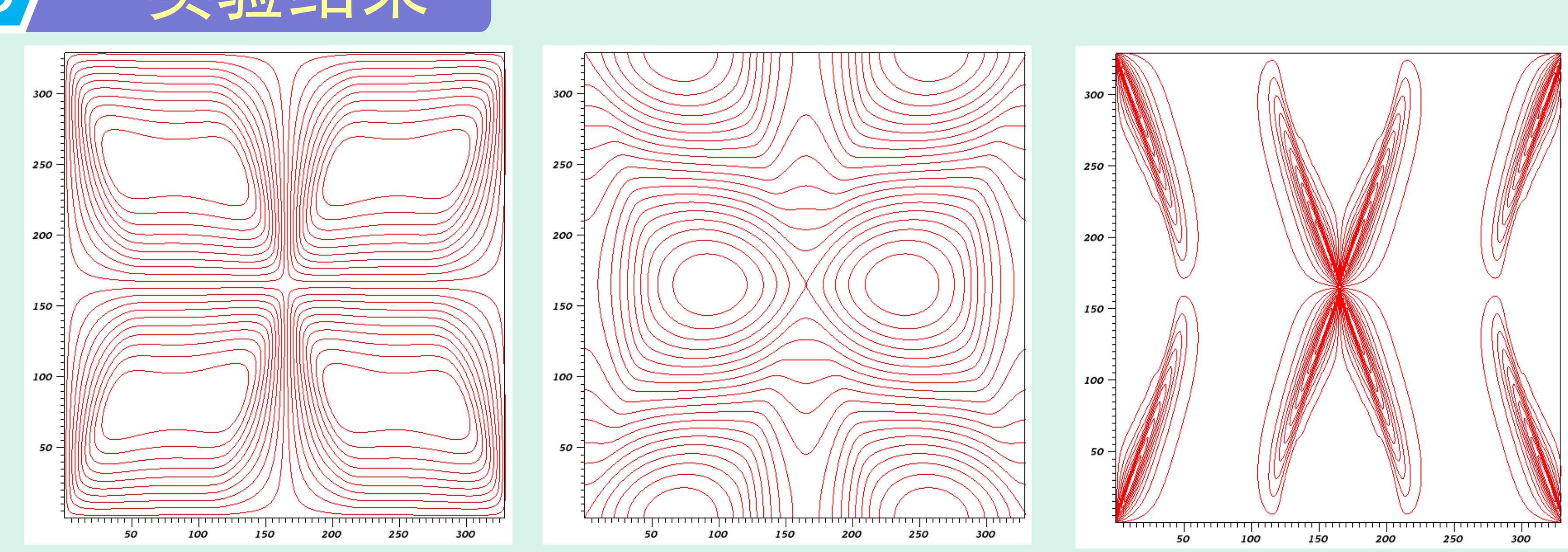


SpMV 任务划分



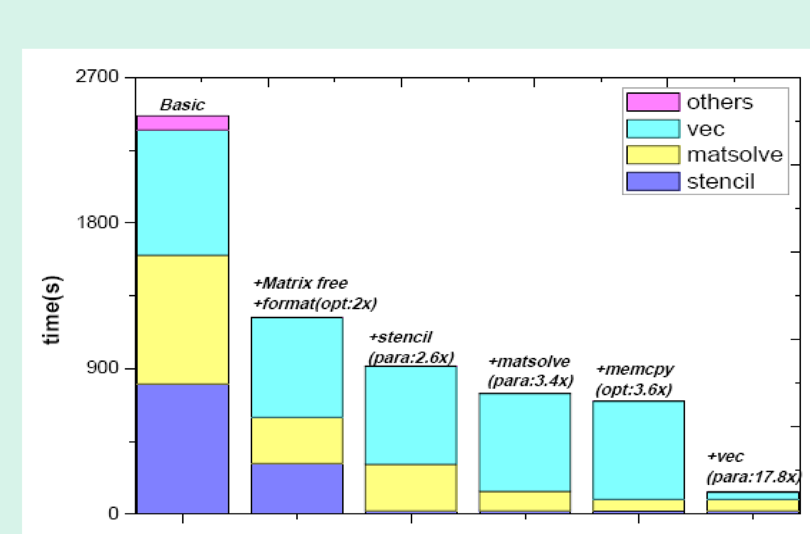
SpMV动静态buffer加载

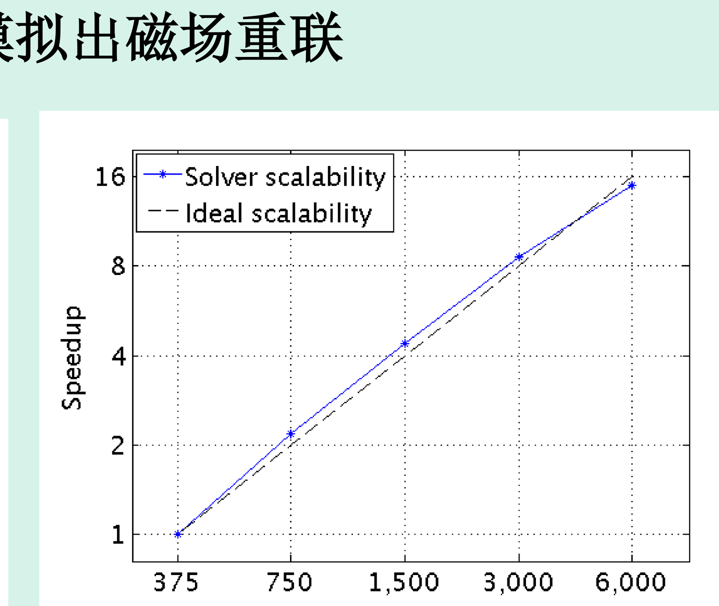
6 实验结果



模拟结果，成功模拟出磁场重联

众核加速效果





强可扩展性