ISCAS 中国科学院软件研究所学术年会'2021 暨计算机科学国家重点实验室开放周 学术论文

Switching Controller Synthesis for Delay Hybrid Systems under Perturbations

干扰下时延混成系统的切换控制器合成

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Proceedings of the 24th International Conference on Hybrid Systems: Computation and Control, HSCC. 2021: 1-11.

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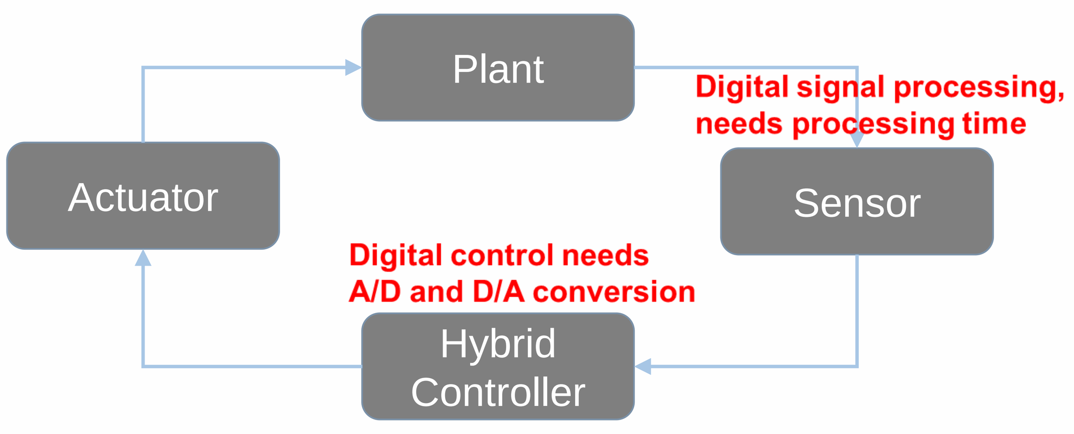
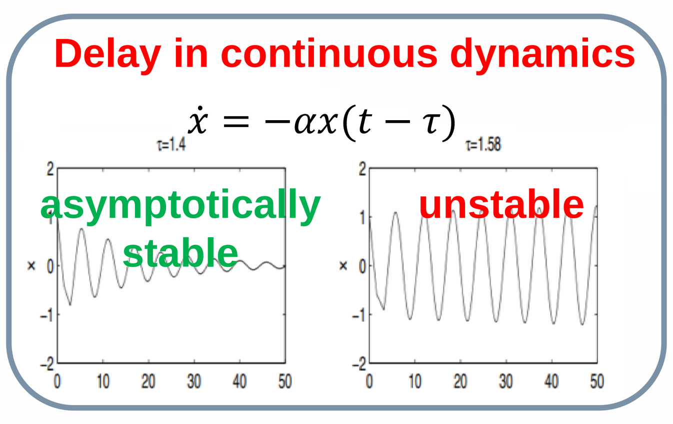
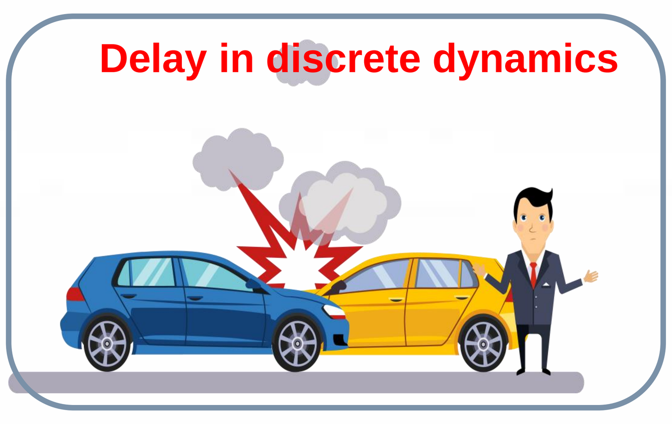
Delay in Safety-Critical Systems

- Many cyber-physical systems are **safety-critical**.
- **Delay** is inevitable!



- **Two kinds of delay** occur in CPS.





Main Contributions

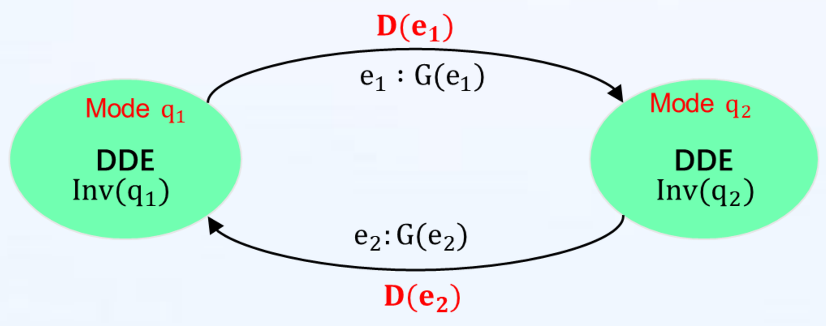
- A new model language, called **delay hybrid automaton**, is proposed to model delay hybrid systems, which exhibit delays in both continuous and discrete-time dynamics.
- A novel approach based on the computation of **differential invariants** is proposed to address the switching controller synthesis problem for delay hybrid systems.

Delay Hybrid Automaton

Definition (Delay Hybrid Automaton, DHA)

A DHA is a tuple $\mathcal{H} = (Q, X, U, Inv, X_0, F, E, D, G, R)$

- U: a set of continuous functionals;
- Inv: an invariant Inv(q) for each mode q ∈ Q ;
- R: E × X_D → U : reset functions;
- ..



Differential Invariant Generation

Non-linear DDE: $\dot{x}(t) = f(x(t), x(t-r), w(t))$

linearize

Linear DDE: $\dot{x}(t) = A\,x(t) + Bx(t-r) + Cw(t) + g(x(t), x(t-r))$

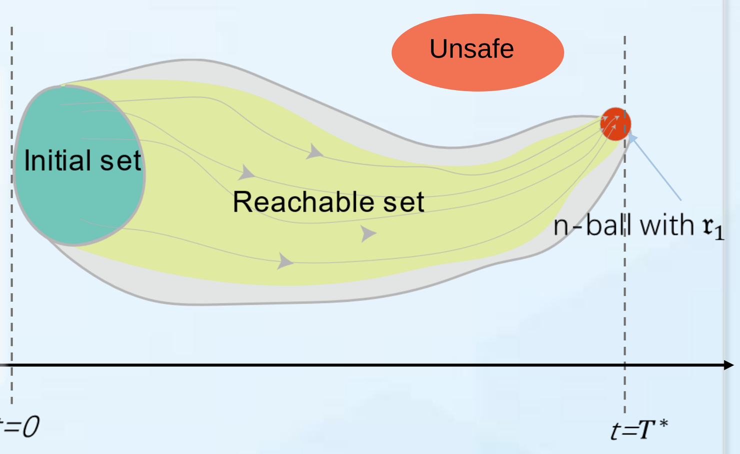
- Reduce to T-invariant, i.e., $\forall T > T^*$, **∞-invariant ⇔ *T*-invariant**

Exponentially convergent to a ball:

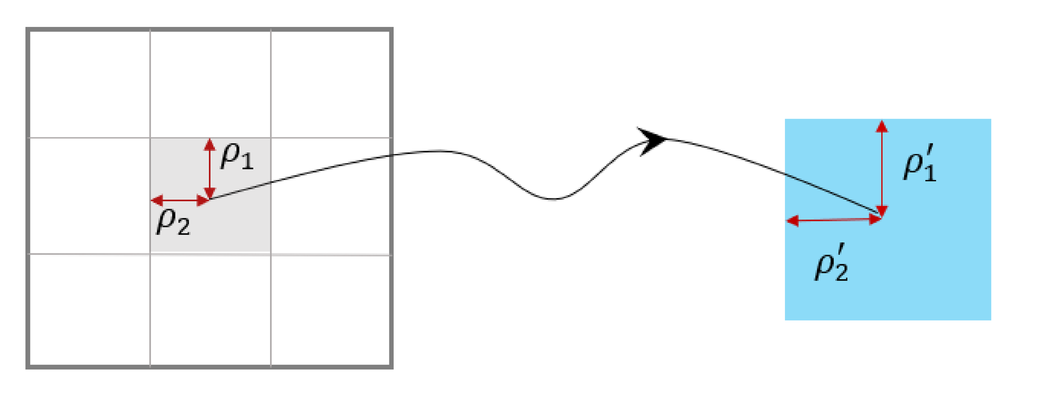
if there exist a constant $\gamma > 0$ and a contant $\beta > 0$ such that

$$\|\xi_\phi^{w}(t)\|_\infty \le r_1 + \beta(\|\phi\|_\infty - r_2)^+ e^{-\gamma t}, \quad \forall t \ge 0$$

holds for all $\phi \in C$, $\|w(t)\|_\infty \le \bar{w}$ $\forall t \ge 0$.

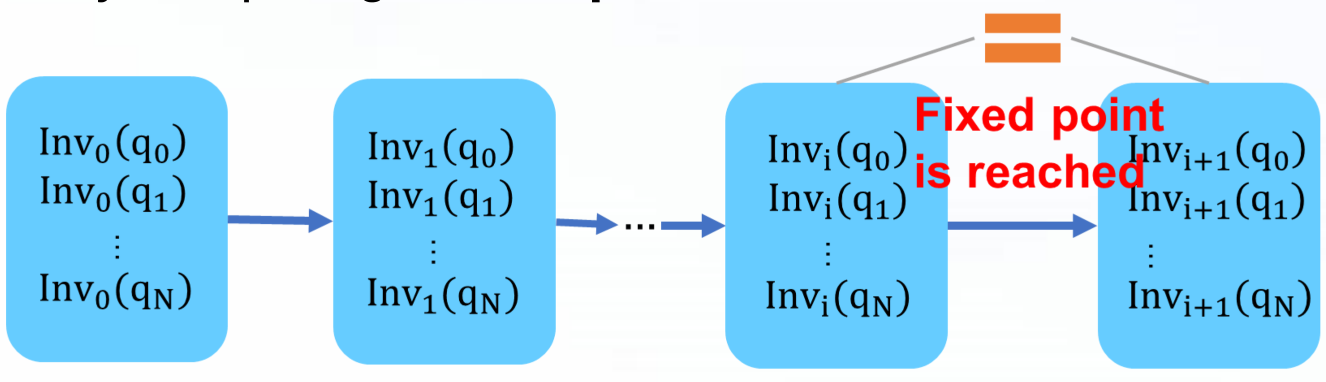


- Compute **a safe over-approximate reachable set** in T

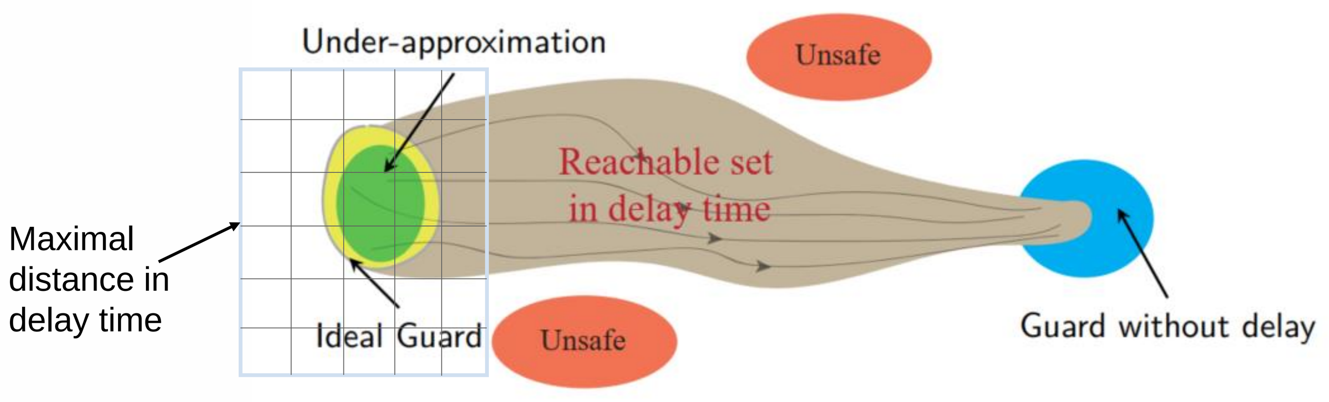


Guard Synthesis under Delay

- Generate a global invariant for delay hybrid system by computing a **fixed point.**



- Synthesize guard condition **without delay** using invariant;
- Synthesize guard condition under delay by **backward reachable set computation.**

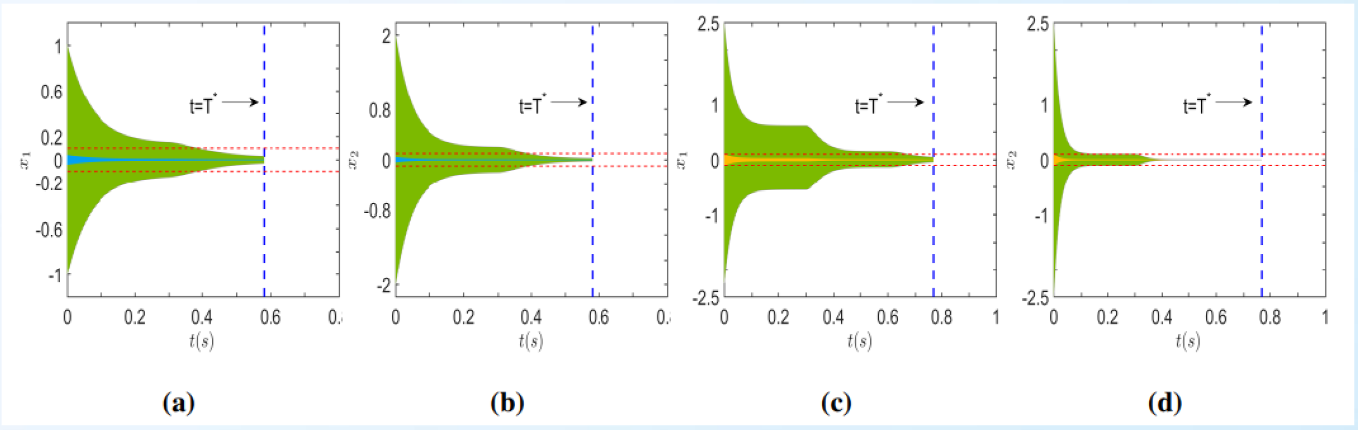


Case Study 1

- **Low-pass Filter System**

$$q_1 : \begin{cases} \begin{cases} \dot{x}_1(t) = -14.58x_1(t) + 2x_1(t-0.1) + 0.5\sin(t) \\ \dot{x}_2(t) = -20.05x_2(t) + 2x_2(t-0.1) + 0.5\sin(t) \end{cases} \\ \Xi(q_1) = [-1, 1] \times [-2, 2] \\ I(q_1) = \mathbb{R}^2, \end{cases}$$

$$q_2 : \begin{cases} \begin{cases} \dot{x}_1(t) = -32.66x_1(t) + 8x_1(t-0.1) + 0.5\sin(t) \\ \dot{x}_2(t) = -47.25x_2(t) + 8x_2(t-0.1) + 0.5\sin(t) \end{cases} \\ \Xi(q_2) = [-2.25, 2.5] \times [-2.5, 2.5] \\ I(q_2) = \mathbb{R}^2. \end{cases}$$



Case Study 2

- **Predator-prey Populations system**

$$q_1 : \begin{cases} \begin{cases} \dot{x}_1(t) = -x_1(t)(1 - \frac{x_1(t)}{100}) + 0.2d_1 + w_{11}(t) \\ \dot{x}_2(t) = -1.5x_2(t)(1 - \frac{x_2(t)}{100}) + 0.1d_2 + w_{12}(t) \end{cases} \\ \Xi(q_1) = [-0.2, 0.2] \times [-0.1, 0.1] \\ I(q_1) = \mathbb{R}^2. \end{cases}$$

$$q_2 : \begin{cases} \begin{cases} \dot{x}_1(t) = -2.5x_1(t) + 0.2x_1(t-0.01)(1 + x_2(t)) + w_{21}(t) \\ \dot{x}_2(t) = -2x_2(t) + 0.15x_2(t-0.01)(1 + x_2(t)) + w_{22}(t) \end{cases} \\ \Xi(q_2) = [-0.2, 0.2] \times [-0.2, 0.2] \\ I(q_2) = \mathbb{R}^2. \end{cases}$$