MONOTONOUS BETTING STRATEGIES IN WARPED CASINOS

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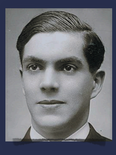
Richard von Mises (1883-1953). Known for his approach to probability via place selections in random outcomes and the impossibility of a gambling system: methodical selection of subsequences of a random sequence does not change the probability of specific elements.



Jean Ville (1910-1989) put Mises' theory on a firm basis through his formalization of betting strategies via martingales; today's game-theoretic probability owes much to him.

Joseph Doob (1910-2004) developed martingales into one of the most powerful tools in modern probability.

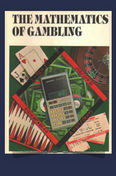
Claus-Peter Schnorr (1943-) used Ville-style martingales to study and calibrate algorithmic randomness.

BETTING SYSTEMS have 2 components:

- the outcome where we bet
- the amount we bet (wager)

STRATEGY is a function mapping finite sequences of outcomes to the amount the player bets at that particular round and state of the game.

Ville's idea was to use functions that satisfy a fairness condition, to model the capital of the gambler during the game as a stochastic process, hence avoiding explicit reference to the underlying strategy function.



FAIRNESS

Fairness means that on average, the expectation of profit is 0. The only way to get profit x with very high probability, is that with very small probability you lose a large amount, compared to x.

MONOTONOUS BETTING

A strategy is monotonous or single-sided if it always bets on the same outcome, possibly varying the wager.

Many known strategies are single-sided: the Martingale, D'Alembert, Oscar Grind, the Paroli, Cancellation System.

The problem of optimal gambling strategies has motivated technical research in probability, largely initiated and based on the monograph of DUBBINS & SAVAGE. Topics include restricted strategies, inflation, income-tax, bold and timid play, lotteries, stopped games.

RECENT DEVELOPMENTS (2010-)

Research on constructive betting strategies, where information accessible to the players may be limited or gradually released was mainly established in the last 10 years. This was motivated by qualifying and quantifying algorithmic randomness through martingales, but also by the concerns of game-theorists with respect to the strength of restricted strategies in two-player zero-sum games.

BIASED (WARPED) CASINOS

Casinos where the player has an edge, so outcomes are not entirely random; known as "favorable games" in the framework of Dubbins & Savage.

RESEARCH PROBLEM

Can we always succeed in a biased casino with a single-sided strategy? If there is a strategy that beats the casino, is there a single-sided winning strategy?

OUTCOME OF OUR RESEARCH

- ❖ When the strategies are programmable, answer is positive.
- ❖ For mixtures of programmable strategies, answer is negative.

In fact, there exists a casino where:

- ‣ a programmable mixture strategy succeeds exponentially fast
- ‣ but no single-sided programmable mixture succeeds at all.

METHODOLOGY

- Every martingale is the product of two opposite-sided martingales
- this decompositions holds constructively, but not for mixtures - strategies with enumerable wager and capital
- enumerable martingales can be written as computable countable mixtures of martingales
- results are generalized to computably-sided strategies, where the favorable outcome is decidable.

LIMITATIONS AND FUTURE WORK

- our methods do not apply to discounted horizons or inflation, represented by supermartingales
- our impossibility result assumes enumerability of not only the capital, but also the wager.
- the general case of unrestricted effective mixtures (enumerable capital) is the topic of future work.